Chain Formation in Ferrofluid

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We showed in previous work that thin layers of ferrofluid consisting of magnetite particles have the property to scatter light along beams so long as the assumption of rod formation holds. The following analysis shows that this assumption is correct. More numerical analysis is certainly possible, but the following is a fairly simple and straight forward method that gives very good comparison to what is observed. That is, light is scattered by what appear to be rod shapes in the Flux Resonator and the forces acting on particles in ferrofluid are consistent with the formation of rod like shapes.

In Rosensweig's "Ferrohydrodynamics" and Jackson's "Classical Electrodynamics" we find the force on a dipole (with no current) to be given by

$$\vec{F} = \left(\vec{m} \cdot \vec{\nabla}\right)\vec{B} \quad (1$$

where $\vec{m}$ is the dipole moment and $\vec{B}$ is the field acting on the dipole. In previous analysis I found the field outside a particle to be

$$\vec{B} = \left[B_0 + \frac{\mu_0\, d^3\, M_s}{4r^3}\left(\frac{1-3^{-a\,B_0}}{1+3^{-a\,B_0}}\right)\right]\cos\theta\,\hat{r} - \left[B_0 + \frac{\mu_0\, d^3\, M_s}{8r^3}\left(\frac{1-3^{-a\,B_0}}{1+3^{-a\,B_0}}\right)\right]\sin\theta\,\hat{\theta} \quad (2$$

Where d is the particle diameter, B_0 is the external applied field, μ_0 is the permibility of free space, $a = 1/B_{1/2}$ is the half way point to saturation, M_s is the saturation magnitude of magnetite and (r, θ) is the distance and angle from the center of the particle.

I also found the field inside the particle. This is given by

$$\vec{b} = \left[B_0 + \mu_0 \, M_s \left(\frac{1 - 3^{-a B_0}}{1 + 3^{-a B_0}} \right) \right] \hat{z} \tag{3}$$

(Note that the form given appears different, but it is really the same thing. A transformation of coordinates has been used to simplify life, and we transform back when we need to. $\hat{z} = \cos\theta \, \hat{r} - \sin\theta \, \hat{\theta}$)

The magnetic dipole moment is defined in Jackson as

$$\vec{m} = \int_V \vec{b} \, d^3x \tag{4}$$

Where V is the volume of the particle. Putting 3 into 4, and using $d^3x = 2\pi r^2 dr \, d\theta$ (since there is no φ component) we find

$$\vec{m} = \frac{\pi d^3}{6} \, \mu_0 \, M_s \left(\frac{1 - 3^{-a B_0}}{1 + 3^{-a B_0}} \right) \hat{z} \tag{5}$$

Using equation 5 and converting to spherical coordinates as noted above we have

$$\left(\vec{m} \cdot \vec{\nabla} \right) = \frac{\pi d^3}{6} \, \mu_0 \, M_s \left(\frac{1 - 3^{-a B_0}}{1 + 3^{-a B_0}} \right) \left[\cos\theta \, \frac{\partial}{\partial r} - \frac{\sin\theta}{r} \frac{\partial}{\partial \theta} \right] \tag{6}$$

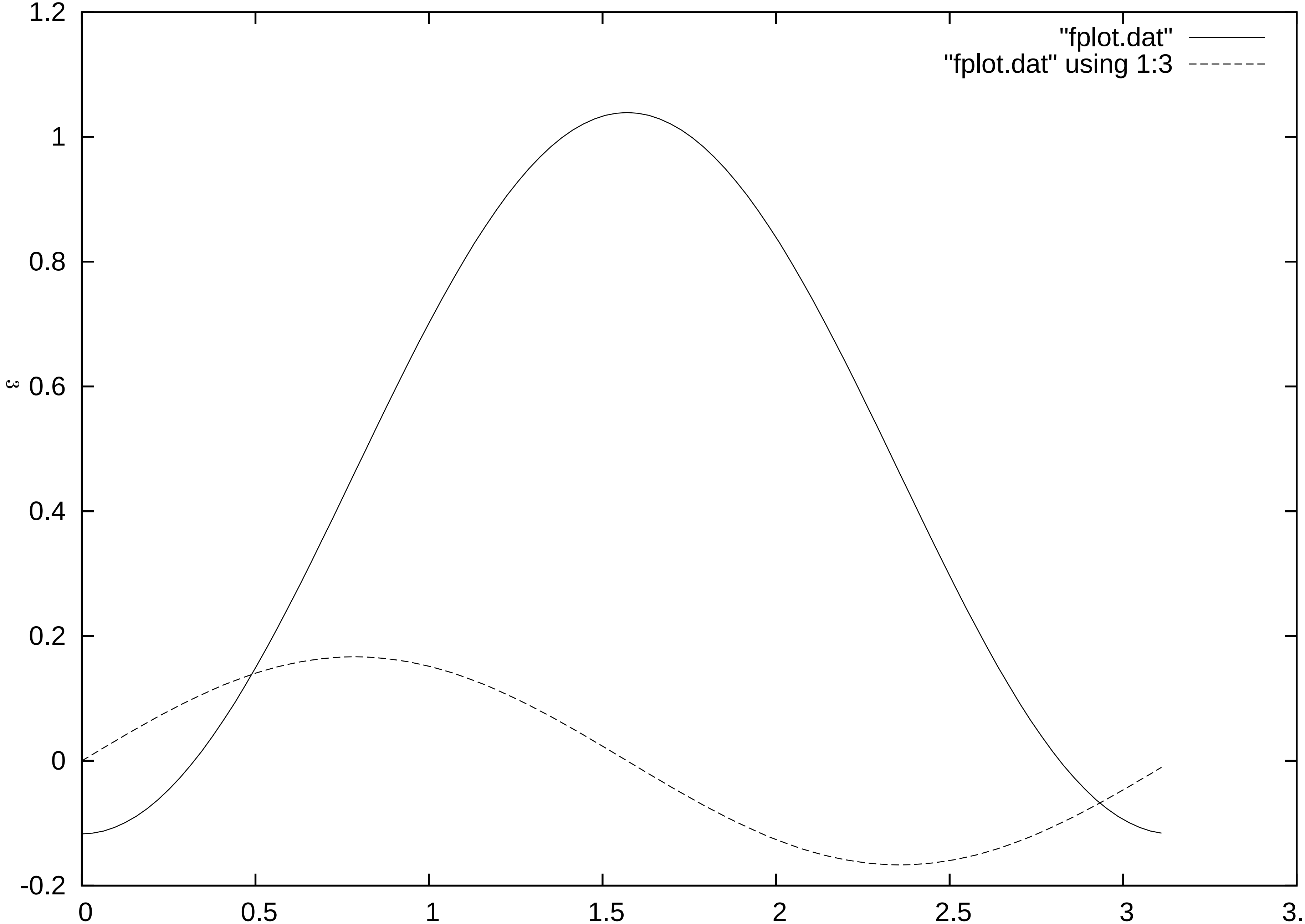
With form 6 we can operate on equation 2 to find the total force $\vec{F}$ acting between two particles in the ferrofluid. To simplify writing let's use

$$M = \frac{\mu_0 \, d^3 \, M_s}{2} \left(\frac{1 - 3^{-a B_0}}{1 + 3^{-a B_0}} \right) \tag{7}$$

Crunching and grinding 6 on 2 we end up with

$$\vec{F} = \frac{\pi M}{r} \left\{ \left[B_0 \sin^2\theta + \frac{M}{r^3} \left(\frac{\sin^2\theta}{3} - \cos^2\theta \right) \right] \hat{r} + \left(B_0 + \frac{M}{r^3} \right) \frac{\sin\theta \cos\theta}{3} \hat{\theta} \right\} \tag{8}$$

Using $d = 10$nm, $r = 14$nm, $M_s = 5.11 \times 10^5 A/m$, $B_0 = 1T$, and $a = 1/0.0158 T^{-1}$ we find the following curves for the force components inside the curly brackets of equation 8.



"fplot.dat"
"fplot.dat" using 1:3
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
0.5
1
1.5
2
2.5
3
3.5

The dark line is the radial force and the dotted line is the tangential force. Both are plotted as a function of angle. It is clear that the radial forces are negative along the axis of the applied field. The tangential force is negative below the particle causing it to be pushed further below, and positive above, causing it to be pushed up. Once the particle falls into the well, it will be radially attracted by the negative force.

It is clear that chain formation must happen, and that chains of single particles will push other chains away. Thus, not only does rod formation happen, the rods will be composed of mostly single particles. This is consistent with observation and calculation from the light scattering analysis.